COLUMBIA | ENGINEERING
The Fu Foundation School of Engineering and Applied Science

ELEN E3106/4106 Lecture 16

BJTs Part II

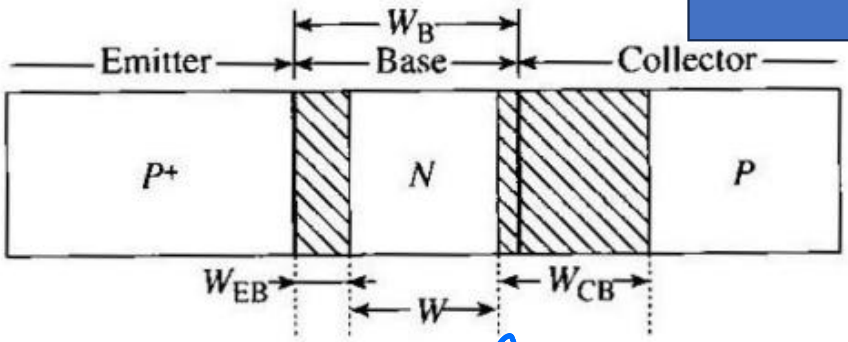
Outline

- Cross-Section of fabricated BJT
- Minority carrier concentrations
- Approximations of current in the 3 regions
- Gummel numbers and plots
- High-level injection & Knee Current
- Current Gain

Assignments:
Reading: Streetman and Banerjee §7.1-7.4
Reading: C. Hu §8.1-8.4

Terminology and Notation (p-n-p)

Note: For n-p-n, the subscripts on the right sides of the equal signs will flip from n to p or vice verse (ex. $D_E = D_p$ in n-p-n) Why? Because we are focusing on the minority carrier types in each region!



Emitter	Base	Collector
I_E	I_B	I_C
W_E	W_B	W_C
$N_E = N_{a,E}$	$N_B = N_{d,B}$	$N_C = N_{a,C}$
$D_E = D_n$	$D_B = D_p$	$D_C = D_n$
$\tau_E = \tau_n$	$\tau_B = \tau_p$	$\tau_C = \tau_n$
$L_E = L_n$	$L_B = L_p$	$L_C = L_n$
$n_E = \frac{n_{iE}^2}{N_E}$	$n_B = \frac{n_{iB}^2}{N_B}$	$n_C = \frac{n_{iC}^2}{N_C}$

Sources: A. Doolittle ECE 3040 Slides

n-p-n

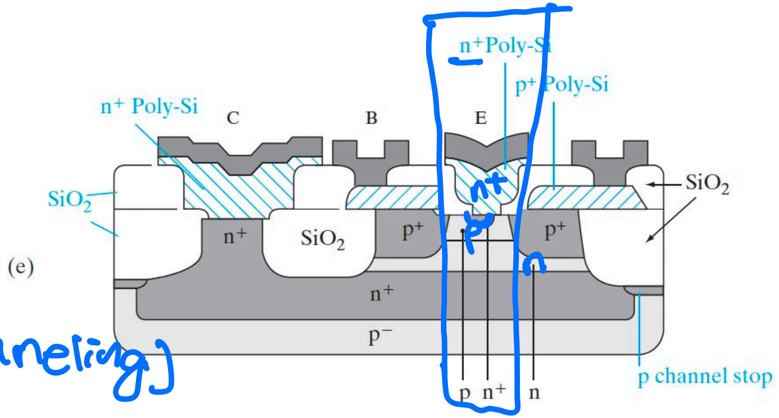
Cross-Section of a Realistic BJT

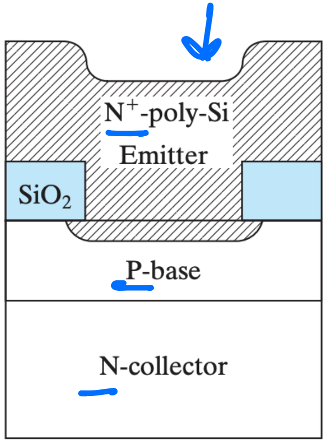
- What type of BJT is this?

  n-p-n

- Why are there extra n+ and p+ regions?
  - Providing good ohmic contacts (tunneling)
  - Reduce parasitic series resistance

- Why do we need the SiO_2?
  - Electrical separation/insulation to separate devices/regions from one another





Sources: Textbook, E. Pop ECE 340 Lecture Slides, C. Hu *Modern Semiconductor Devices for Integrated Circuits*

Assumptions for Simplified BJT Analysis

p-n-p

- Back to our simplified analysis of a p-n-p. We will assume:
  1. Holes diffuse from E → C (ignore drift)
  2. I_E made up entirely of holes (emitter injection efficiency $\gamma = 1$)
  3. Collector saturation current is negligible
  4. Ignore recombination in SCR (for now, then add it back in)
  5. All currents and voltages are steady-state
  6. Cross-sectional area of both junctions is uniform and current flow is essentially 1D

- What ratio of dimensions constitute narrow (short) vs. long base?
  - Short: $W_B \leq 3L_B$
  - Long: $W_B > 3L_B$

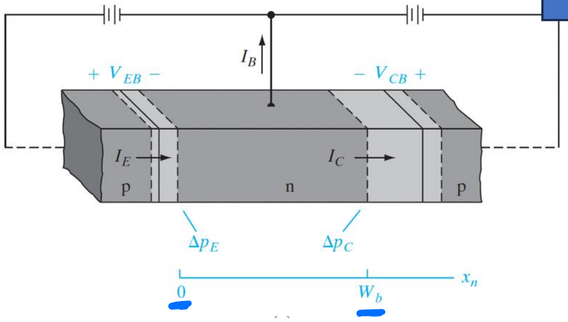
Sources: Textbook

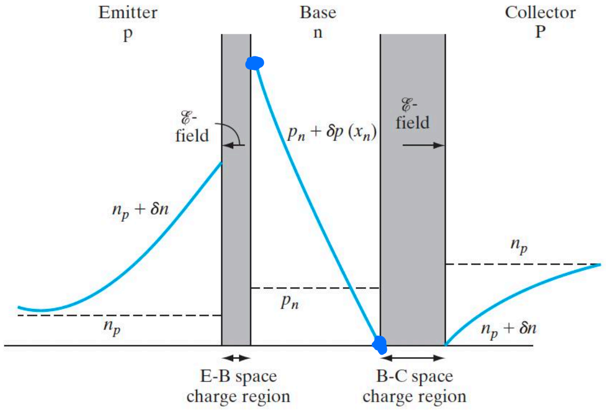
Narrow Base Minority Carrier Concentrations

- +x-dir: Current entering base: I_E; current leaving base: I_C
- A is cross-sectional area, W_B is width of the base
- Recalling our equation for excess minority carrier concentrations injected at the edge of the depletion region ($\Delta p_n = p_n\left(e^{\frac{qV}{kT}} - 1\right)$), we can write the excess h+ concentrations at the edge of the emitter and collector depletion regions:

$$\Delta p_E = p_n(e^{qV_{EB}/kT} - 1) \simeq p_n e^{qV_{EB}/kT}$$
$$\Delta p_C = p_n(e^{qV_{CB}/kT} - 1) \simeq -p_n$$

- Which simplify when EB is strongly FB ($V_{EB} \gg kT/q$) and CB is strongly RB ($V_{CB} \ll 0$)
- Also recall we denote excess carrier concentrations as a function of distance $\delta p(x_n)$ where $\delta p(0) = \Delta p_E$; $\delta p(W_b) = \Delta p_C$





Sources: Textbook, E. Pop ECE 340 Lecture Slides

Approximation of Collector Current

- Now, we can *almost* find the collector (hole) current (assuming diffusion):

$$I_C \approx -qAD_p \left.\frac{dp}{dx}\right|_{x=W_B}$$

- From our discussion of narrow-base diodes ($W_b \ll L_p$), $\frac{dp}{dx}$ varies linearly with the straight line approximation, and the slope is:

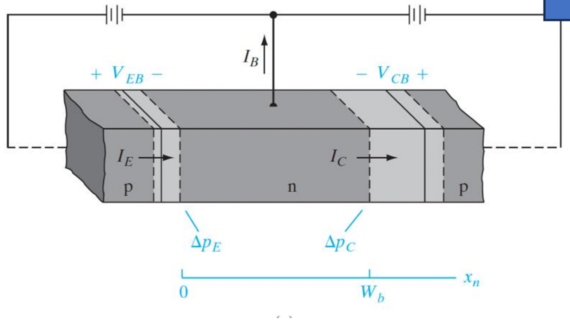
$$\frac{dp}{dx} \approx \frac{-(\Delta p_E + \Delta p_C)}{W_B} \approx \frac{-p_n}{W_B}\left(e^{\frac{qV_{EB}}{kT}} - 1\right)$$

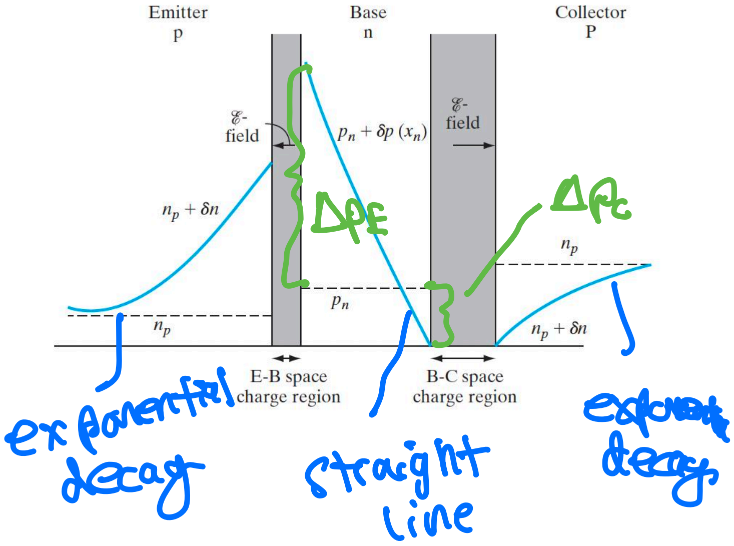
- Then, our current becomes:

$$I_C \approx \boxed{qA\frac{D_B}{W_B}\frac{n_i^2}{N_B}}\left(e^{\frac{qV_{EB}}{kT}} - 1\right)$$

- Which we can rewrite in terms of the saturation current, I_S

$$I_C = I_S\left(e^{\frac{qV_{EB}}{kT}} - 1\right)$$





Sources: Textbook, E. Pop ECE 340 Lecture Slides, C. Hu *Modern Semiconductor Devices for Integrated Circuits*

Base Gummel Number

p-n-p

- We can rewrite the collector curent $I_C = I_S(e^{\frac{qV_{EB}}{kT}} - 1)$, as $I_C = A\frac{qn_i^2}{G_B}(e^{\frac{qV_{EB}}{kT}} - 1)$

n_{iB} is the intrinsic carrier concentration of the base material. The subscript, B, is added to n_i because the base may be made of a different semiconductor than the emitter and collector material!

- Where G_B, the base gummel # (units: s/cm^4) is:

$$G_B = \frac{n_i^2}{n_{iB}^2}\frac{N_B}{D_B}W_B = \frac{n_i^2}{n_{iB}^2}\frac{n}{D_B}W_B$$

- The more general definition valid even for non-uniform doping in the base and high-level conditions:

$$G_B \equiv \int_0^{W_B} \frac{n_i^2}{n_{iB}^2}\frac{n}{D_B}dx$$

- In the special case where $n_i = n_i^B$, D_B is constant, and low-level injection ($p(x) = N_B(x)$):

$$G_B = \frac{1}{D_B}\int_0^{W_B} N_B(x)dx = \frac{1}{D_B} \times \text{base dopant atoms per unit area}$$

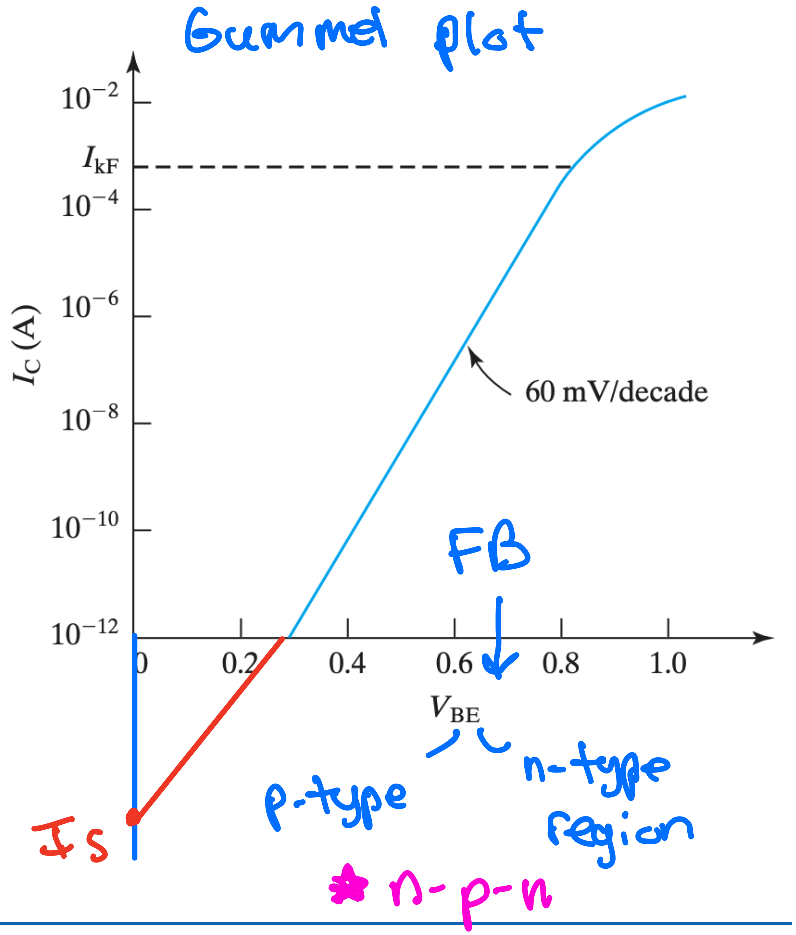
Sources: C. Hu *Modern Semiconductor Devices for Integrated Circuits*

Gummel Plots

n-p-n. How can we tell? Plot in terms of V_{BE} instead of ____ !

- So, now we know the base gummel number is basically proportional to the base dopant density per area
- The higher the base dopant density, the lower I_C will be for a given V_{BE}
- G_B contains all of the subtleties of transistor design that impact I_C
- And… G_B can be easily experimentally determined from the Gummel plot (semilog of I_C and sometimes I_B versus VBE)

1. Extrapolate straight line to find the y-intercept. This is I_S.
2. $G_B = Aqn_i^2/I_S$



Sources: C. Hu *Modern Semiconductor Devices for Integrated Circuits*

High Level Injection and Knee Current

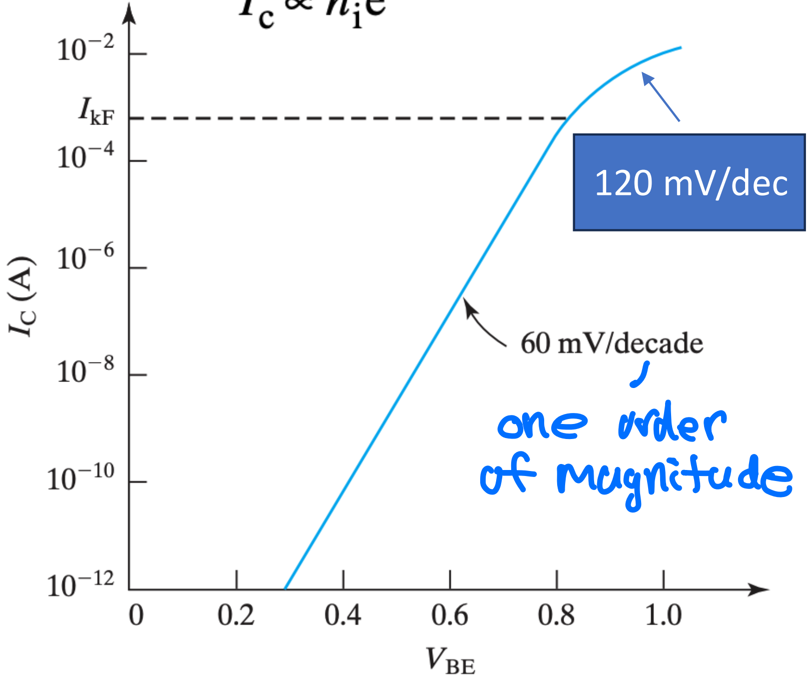
n-p-n

- Why does the slope decrease at high I_C?
- At large V_{BE}, $\delta n(x_p)$ in the base can become larger than the base doping concentration: $\delta n(x_p) = \delta p(x_p)$ $\gg N_B$
- Recall in H.L.I., minority AND majority carrier concentrations are changed substantially in base region
- Thus, the inverse slope is increased by a factor of 2 and becomes 120 mV/dec
- The current at which the slope changes is called the knee current, I_{kF}

$$n \approx p \approx n_i e^{\frac{qV}{2kT}}$$

$$G_B \propto n_i e^{qV_{BE}/2kT}$$

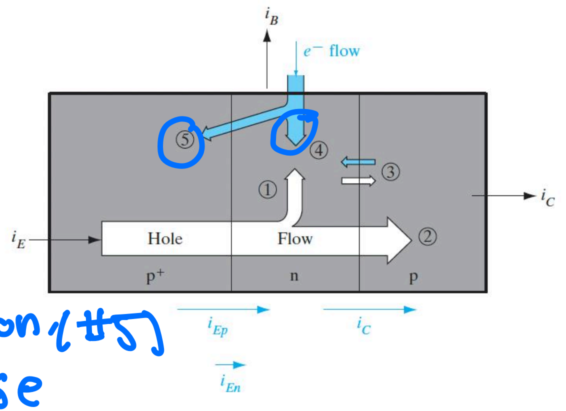
$$I_c \propto n_i e^{qV_{BE}/2kT}$$



Sources: C. Hu *Modern Semiconductor Devices for Integrated Circuits*

Approximation of Base Current

*1 A = 1 C/s



p-n-p

What about the base current?

- From our last discussion on I_B, base current exists primarily due to e- which ← p-n-p
  - (a) are injected into the emitter across the FB biased EB junction (#5)
  - (b) recombine with a small fraction ( ) of the h+ transiting the base

$$I_B = I_B(\text{inj. to E}) + I_B(\text{recomb. with excess holes})$$ #4

#5 #4

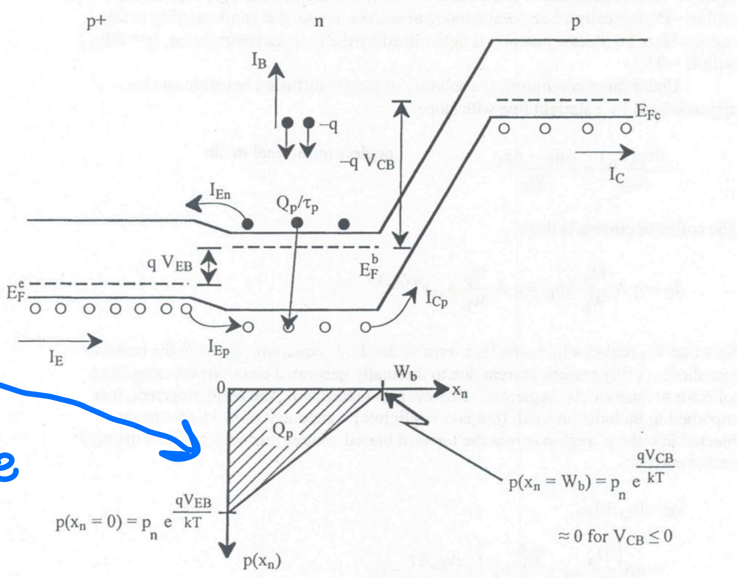
$$I_B \approx I_{En} + Q_p/\tau_p$$ (C/s)

$$I_B \approx I_{En} + \boxed{\frac{1}{2\tau_p} A W_B p_n \left(e^{\frac{qV_{EB}}{kT}} - 1\right)}$$

- The current gain can then be written:

$$\beta = \frac{I_C}{I_B} \approx \frac{I_{Ep}}{I_{En} + I_{B,rec.}}$$

area under minority carrier conc. curve



Sources: http://transport.ece.illinois.edu/ECE340S16-Lectures/NarrowBaseDiode.pdf

Emitter Gummel Number and Base Current

n-p-n

- Conveniently, we can express the base current in terms of another Gummel number the emitter gummel # , GE

$$I_B = A_E \frac{q n_i^2}{G_E}\left(e^{qV_{BE}/kT} - 1\right)$$

$$G_E = \int_0^{W_E} \frac{n_i^2}{n_{iE}^2} \frac{n}{D_E} dx$$

Similarly, n_{iE} is the intrinsic carrier concentration of the emitter material. The subscript, E, is added to n_i because the base may be made of a different semiconductor than the emitter.

- In the case where the emitter is uniformly doped, and D_E is constant:

$$I_B = A_E q \frac{D_E}{W_E} \frac{n_{iE}^2}{N_E}\left(e^{qV_{BE}/kT} - 1\right)$$

Sources: C. Hu *Modern Semiconductor Devices for Integrated Circuits*

Expressions for Current Gain

p-n-p

β can be simplified in 2 ways:

1. If we assume no recombination in the base,

$$\beta(no\ recomb.) \approx \frac{I_C}{I_{En}} \approx \frac{D_B L_E N_E}{D_E W_B N_B}$$

2. If emitter injection efficiency is perfect, $\gamma = 1$, $I_{B,rec.} \gg I_{B,inj.}$ and

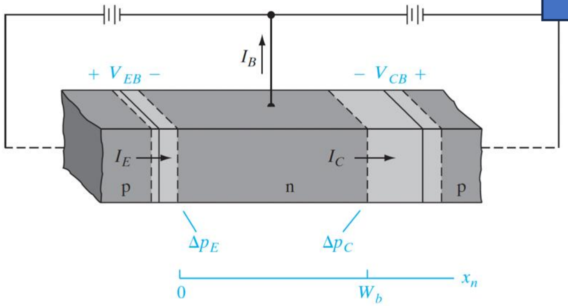
$$\beta(\gamma \to 1) \approx \frac{I_C}{I_{B,rec.}} \approx 2\left(\frac{L_B}{W_B}\right)^2$$

Where $L_B^2 = D_B \tau_B$, and $W_B \ll L_B$

- Where did the square and factor of 2 come from? empirically true
- How do you find $D_{E,B}$? Recall Einstein Relation: $\frac{D}{\mu} = \frac{kT}{q}$

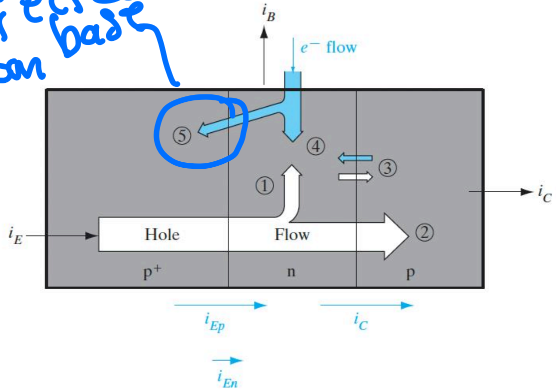
Sources: E. Pop ECE 340 Lecture Slides, http://transport.ece.illinois.edu/ECE340S16-Lectures/NarrowBaseDiode.pdf

Approximation of Emitter Current



- Now, we can find the emitter (hole) current (diffusion only), considering there is a small e⁻ injection component from the n-type base (#5 on diagram)
- If emitter is long ($W_E \gg L_E$) minority carrier concentration exponentially decays. If emitter is short, $(W_E \ll L_E)$, straight line approximation applies.
- If emitter is long, expression for long-base diode: typically

injected e⁻ from base



$$I_{En} = qA\frac{D_n^E}{L_n^E}\frac{n_i^2}{N_E}\left(e^{qV_{EB}/kT} - 1\right)$$

- We can finally write total emitter current,

$$I_E = I_{Ep} + I_{En} \approx I_C + I_{En} \approx qAn_i^2\left[\frac{D_p^B}{W_B N_B} + \frac{D_n^E}{L_n^E N_E}\right]\left(e^{\frac{qV_{EB}}{kT}} - 1\right)$$

And, now we can calculate the FOM emitter injection efficiency, $\boxed{\gamma = I_{Ep}/I_E}$!

Sources: Textbook, E. Pop ECE 340 Lecture Slides

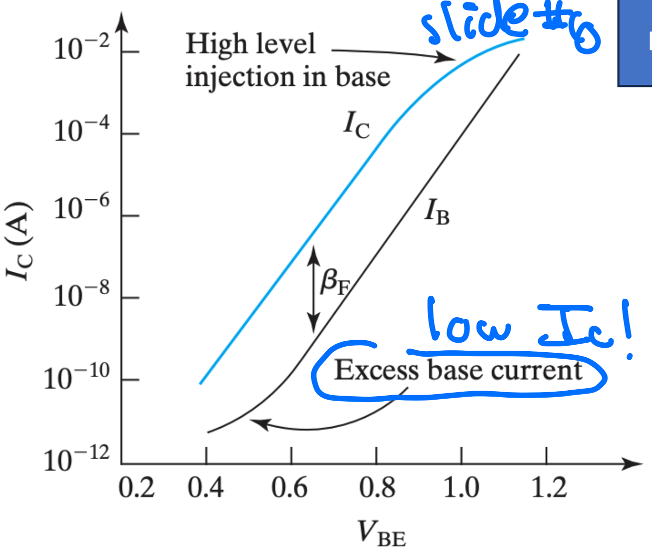
Gummel Plot and β Fall-Off at High and Low I_C

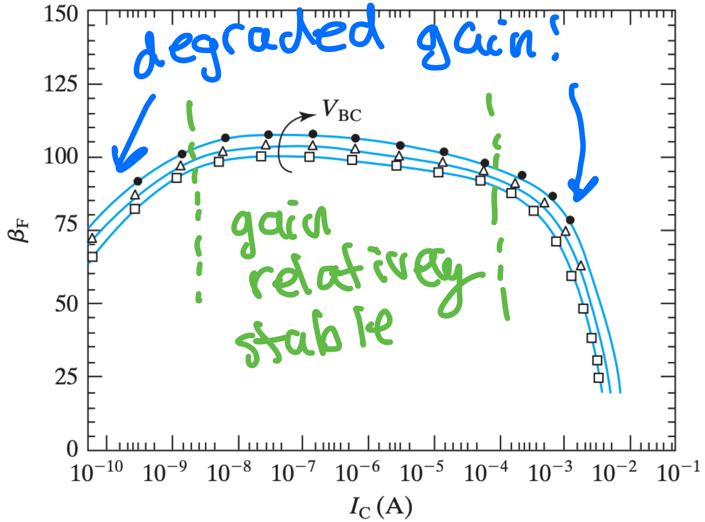
n-p-n

- Slide 10 recap: High I_C (used for high-speed)
  - High level injection effect in the base flattens I_C
  - I_B injected into emitter is unaffected because emitter is very heavily doped (it's practically impossible to inject more h+ than N_E)
- Low I_C (used for low-power circuits)
  - Non-idealities such as gen.-recomb in the EB SCR can dominate at low current levels

$$I_B = I_B(\text{inj. to E}) + I_B(\text{recomb. with excess holes})$$

  - Recall we accounted for this with the diode ideality factor, n
- Summary: Current gain, $\beta = \frac{I_C}{I_B}$ is degraded in both high, low I_C cases





Sources: C. Hu *Modern Semiconductor Devices for Integrated Circuits*

Transit Time and Charge Storage

p-n-p

- By design, $\beta \gg$ 1 for current *amplification*. Let's discuss the essential features of current amplification effects (neglecting $I_{B,inj.}$)
- In a properly designed amplification circuit, I_B can be modulated independently
- The junction voltages are not clamped and will adjust themselves accordingly to changes I_B
- Due to space charge neutrality, the stored minority hole charge is determined by the small I_B:

$$Q_p = I_{B,rec.}\tau_p$$

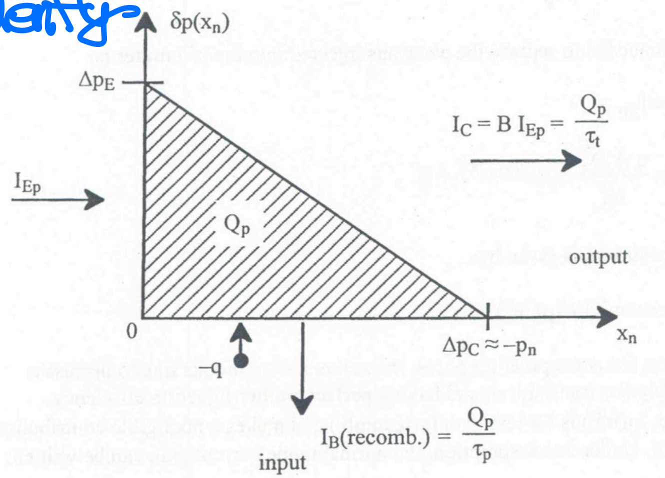
carrier lifetime

- I_C depends on the stored charge, and an average transit time, τ_t, which is much less than the average recombination lifetime ($\tau_t \ll \tau_p$)

$$I_C = Q_p/\tau_t$$

$$\beta(\gamma \to 1) \approx \frac{I_C}{I_{B,rec.}} \approx 2\left(\frac{L_p}{W_B}\right)^2 = \frac{\tau_p}{\tau_t} \gg 1$$

- Transit time: average time it takes for excess carrier to move from emitter to collector
- Charge storage and transit time are crucial factors in determining the speed and frequency response of a BJT, as we will see



Base region, n-type

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